Gaussian Elimination and Row-Echelon Form Linear Algebra X. Du

- **Gaussian Elimination**
  - Method of finding the solution to a system of linear equations with an augmented matrix. We aim to reduce the augmented matrix to reduced row-echelon form.
  - Legal operations:
    - Multiply rows by a constant (cannot be 0).
    - Switch the rows.
    - Add rows.
  - These operations are the same as applying any elementary matrix as a transformation on the associated augmented matrix.
  - Steps:
    - 1. Locate the leftmost column not $\vec{0}$
    - 2. If needed, switch a row so that a non-zero number occupies the first entry of that column.
    - 3. Multiply the top row by the reciprocal of the first non-zero entry so that the row's first non-zero entry is a 1. This entry is called a **pivot**.
    - 4. For each successive row, add multiples of the top row so that all entries below the leading 1 become zeros.
    - 5. Repeat steps 1 through 4 for each successive row, ignoring rows above that you have already reduced. Upon the completion of this step, your matrix should be in row-echelon form.
    - 6. Find the last row not $\vec{0}$. For each previous row going upwards, add multiples of successive rows so that all entries above the leading 1's are 0. Upon the completion of this step, your matrix should be in reduced row-echelon form.
- **Row-Echelon Form**
  - Properties:
    - If the row is not $\vec{0}$:
      - The first non-zero entry of that row is called a **pivot**.
      - All successive rows not $\vec{0}$ have pivots further to the right (i.e. all entries below a pivot entry should be 0).
    - If the row is $\vec{0}$, then it is placed at the bottom of the matrix.
    - Example: $\begin{pmatrix} 2 & 3 & 6 & 1 \\ 0 & 0 & 9 & 2 \\ 0 & 0 & 0 & 4 \end{pmatrix}$
- **Reduced Row-Echelon Form**: Columns with a pivot have an entry of 1 as the pivot and non-pivot entries as zeros elsewhere.
  - Example: $\begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{pmatrix}$
    - This means that the solution to the original system of linear equations is $x = a$, $y = b$, $z = c$